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NUMERICAL SIMULATIONS OF BINARY NEUTRON STAR MERGERS

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Abstract

Tidal deformity is assumed to be very important parameters in compact astrophysical objects such as binary neutron stars and black holes. Different kinds of tidal deformity for the binary neutron star mergers have been investigated and relevant numerical simulations have been implemented using mathematica coding.

Keywords: *binary neutron star mergers, tidal deformity, mathematica coding*

Introduction

As neutron stars are massive and compact astrophysical objects, the coalescence of binary neutron star systems is one of the most promising sources of gravitational waves observable by ground-based detectors. The gravitational wave signals emitted during a neutron star merger depend on the behavior of neutron star matter at high densities. So, the detection of gravitational waves opens the possibility to constraint the nuclear matter parameters characterizing the EoS[1]. A significant signature carried by gravitational waves is the tidal deformability of the neutron star and it is well explored analytically. In a coalescing binary neutron star system, during the last stage of inspiral, each neutron star develops a mass quadrupole due to the extremely strong tidal gravitational field induced by the other neutron star forming the binary. The dimensionless tidal deformability describes the degree of deformation of a neutron star due to the tidal field of the companion neutron star and is sensitive to the nature of the equation of state (EOS). This research will firstly study about mass for observable merging neutron star systems, then, tidal deformability and finally tidal deformability of binary neutron star. Besides, *c* and *G* are taken by 1 in this research.

Mass for merger neutron star systems

The future investigation of neutron stars merger such as GW170817 will have the factors of masses and spins like to those of known double neutron star systems. Known systems contain at least one pulsar and their masses and spins have been determined by pulsar timing. The total mass $M_T = M_1 + M_2$ is known with precision. For the former systems, it is straightforward to determine q and *M*. However, even in the latter cases, some information about *M* and *q* can be established, using the theoretical model that the minimum neutron star mass is $\geq 1.1 M_{\odot}$. Note that one can write the chirp mass

$$M = \frac{M_1^{3/5} M_2^{3/5}}{M_T^{1/5}}$$

$$M = M_T^{2/5} \left(\frac{M_1}{M_T} \right)^{\frac{3}{5}} M_2^{3/5}$$

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$$M = M_T^{2/5}\left(\frac{M_T - M_2}{M_T}\right)^{\frac{3}{5}} M_2^{3/5}$$

$$M = M_T^{2/5}\left(1 - \frac{M_2}{M_T}\right)^{\frac{3}{5}} M_2^{3/5}, \tag{1}$$

$$q = \frac{M_2}{M_1} = \frac{M_2}{M_T - M_2} \tag{2}$$

so the constraint $1.1M_{\odot} \le M_2 \le M_T/2$ will determine $M(q)$. When M_2 is though smaller than or equal to M_1, the binary mass ratio $q \le 1$.

Tidal deformability of neutron star

Consider a static, spherically symmetric star of mass M retained in a static external quadrupolar tidal field ε_{ij} with the response a quadrupole moment Q_{ij}. In the local asymptotic rest frame (asymptotically mass-centered Cartesian coordinates) of the star with large r the metric coefficient g_{tt} is given by

$$\frac{(1-g_{tt})}{2} = -\frac{M}{r} - \frac{3Q_{ij}}{2r^3}\left(n^i n^j - \frac{1}{3}\delta^{ij}\right) + O\left(\frac{1}{r^3}\right) + \frac{1}{2}\varepsilon_{ij}x^i x^j + O\left(r^3\right) \tag{3}$$

where $n^i = x^i/r$; this expansion defines ε_{ij} and Q_{ij}. That Q_{ij} is connected to the density perturbation $\delta\rho$ in the Newtonian limit by

$$Q_{ij} = \int d^3x\,\delta\rho(x)\left(x_i x_j - \frac{1}{3}r^2\delta_{ij}\right), \tag{4}$$

and ε_{ij} is assumed in terms of the external gravitational potential Φ_{ext} as

$$\varepsilon_{ij} = \frac{\partial^2\Phi_{ext}}{\partial x^i \partial x^j} \tag{5}$$

To linear order in ε_{ij}, the induced quadrupole will be of the form

$$Q_{ij} = -\lambda\varepsilon_{ij} \tag{6}$$

The tensor multipole moments Q_{ij} and ε_{ij} can be decomposed as

$$\varepsilon_{ij} = \sum_{m=-2}^{2}\varepsilon_m y_{ij}^{2m}, \tag{7}$$

$$Q_{ij} = \sum_{m=-2}^{2} Q_m y_{ij}^{2m}, \tag{8}$$

where the symmetric traceless tensors y_{ij}^{2m} are defined by (Thorne1980)

$$y_{2m}(\theta,\varphi) = y_{ij}^{2m} n^i n^j \tag{9}$$

with $n = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$. Thus, equation (6) can be written as

$$Q_m = -\lambda \varepsilon_m \tag{10}$$

Without loss of generality, only one ε_m can assumed no vanishing, this is enough to compute λ.

Here λ is a constant related to the $l = 2$, tidal Love number (apsidal constant) k_2 by

$$k_2 = \frac{3}{2}\lambda R^{-5} \tag{11}$$

Here R is the radius of neutron star and the constant of proportionality λ is the tidal deformability of the neutron star. It measures the magnitude of the quadrupole moment induced by an external tidal field and is proportional to the (dimensionless) $l = 2$ tidal Love number

$$k_2 = \frac{3}{2}\lambda R^{-5} \tag{12}$$

To find the tidal deformability λ from Eq. (12), $k_2$must be firstly calculated by using the method described by Hinderer: A perturbation of the spherically symmetric background metric

$$g = -e^{2\nu}dt^2 + \frac{1}{1-2m/r}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right), \tag{13}$$

In the Regge-Wheeler gauge, $\nu(r)$ is determined by

$$\left(1-\frac{2m}{r}\right)\frac{d\nu}{dr} = \frac{1}{r^2}\left(m + 4\pi r^3 p\right), \tag{14}$$

with δg a linear, quadrupolar, static, polar parity perturbation given by

$$\delta g = \left(-e^{2\nu}dt^2 + \frac{1}{1-2m/r}dr^2\right)Hy_{2,m}(\theta,\phi) + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)Ky_{2,m}(\theta,\phi), \tag{15}$$

where H and K are both functions of r. The perturbed Einstein equation gives a differential equation for H:

$$0 = \frac{d^2H}{dr^2}\left(1-\frac{2m}{r}\right) + \frac{dH}{dr}\left[\frac{2}{r} - \frac{2m}{r^2} + 4\pi r(p-\varepsilon)\right] - H\left[\frac{6}{r^2} - 4\pi\left(5\varepsilon + 9p + \frac{\varepsilon + p}{dp/d\varepsilon}\right) + 4\left(1-\frac{2m}{r}\right)\left(\frac{d\nu}{dr}\right)^2\right]. \tag{16}$$

In vacuum, H can be written as a linear combination of $P_2^2(r/M-1)$ and $Q_2^2(r/M-1)$ where P_2^2 and Q_2^2 are the $l = m = 2$ associated Legendre functions. When expanded in powers of $M = r$ at infinity, $P_2^2(r/M-1) = O(M/r)^3$ and $Q_2^2(r/M-1) = O(r/M)^2$. The coeffcient of P_2^2 is therefore related to the quadrupole moment of the star, and the coeffcient of Q_2^2 is related to the tidal field applied by the neutron star. By matching *H(r)* and its derivative across the surface of the star, one can show

$$\begin{aligned} k_2 = {} & \frac{8C^5}{5}\left(1-2C^2\right)\left[2+2C(y-1)-y\right]\{2C\left[6-3y+3C\times(5y-8)\right] \\ & +4C^3\left[13-11y+C(3y-2)+2C^2(1+y)\right]+3\left(1-2C^2\right)\left[2-y+2C(y-1)\right]\log(1-2C)\}^{-1} \end{aligned} \tag{17}$$

where $C = M/R$ is the compactness of the star and the quantity of $y = y(R)$ can be numerically integrated and evaluate y at the surface of the star.,

$$y(r) = r\frac{H'(r)}{H(r)}, \tag{18}$$

which gives rise to the frst-order differential equation

$$r\frac{dy(r)}{dr} + y(r)^2 + y(r)F(r) + r^2Q(r) = 0 \tag{19}$$

with
$$F(r) = \frac{r - 4\pi r^3(\varepsilon(r) - p(r))}{r - 2m(r)} \tag{20}$$

$$Q(r) = \frac{4\pi r\left(5\varepsilon(r) + 9p(r) + \dfrac{\varepsilon(r) + p(r)}{\partial p(r)/\partial\varepsilon(r)} - \dfrac{6}{4\pi r^2}\right)}{r - 2m(r)} - 4\left[\frac{m(r) + 4\pi r^3 p(r)}{r^2(1 - 2m(r)/r)}\right]^2 \tag{21}$$

To find $y = y(R)$, Eq. (19) can be numerically integrated and evaluate y at the surface of the star. where $m(r)$ is mass enclosed within the radius r, $\varepsilon(r)$ and $p(r)$ are, the energy density and pressure respectively in terms of radial coordinate r of a star. Then, by using polytrope equation of state, numerical integration Love number of Eq. (17) is shown in Figure (1). To pronounce the stellar equation of state (EOS), these quantities are calculated within the nuclear matter model chosen. For a given equation of state (EOS), Eq. (19) can be integrated together with the Tolman-Oppenheimer-Volkoff equations with the boundary conditions $y(0) = 0$, $p(0) = p_c$ and $m(0) = 0$, where $y(0)$, p_c and $m(0)$ are the dimensionless quantity, pressure and mass at the center of the NS, respectively. The dimensionless tidal deformability can be defined as

$$\Lambda \equiv \frac{\lambda}{m^5} = \frac{1}{m^5}\frac{2}{3}k_2R^5 = \frac{2}{3}k_2\frac{R^5}{m^5} = \frac{2}{3}k_2C^{-5} \tag{22}$$

R and M are the radii and masses of the binary components, respectively. k_2 can be readily determined from a first-order differential equation simultaneously integrated with the two usual TOV structural equations and has values ranging from about 0.05 to 0.15 for neutron stars. So, the visualization figure of dimensionless tidal deformability Λ for TOV equation is shown in Figure (2). For black holes, $k_2 = 0$. The tidal deformations of the neutron stars result in excess degeneracy of orbital energy and speed up the final stages of the inspiral. Tidal deformations act oppositely to spin effects, which tend to be more important during earlier stages of the observed gravitational wave signal. Then, the tidal deformability of the neutron stars present in the binary neutron star system can be combined to yield the weighted average as,

$$\tilde{\Lambda} = \frac{16}{13}\frac{(12q+1)\Lambda_1 + (12+q)q^4\Lambda_2}{(1+q)^5} \tag{23}$$

where Λ_1 and Λ_2 are the individual tidal deformabilities corresponding to the two components in the NS binary with masses m_1 and m_2, respectively with $q = m_2/m_1 < 1$.

Polytrope equation of state and Tidal deformability of binary neutron star

To investigate impressive a common EOS constraint, a piecewise polytrope scheme is employed to simulate thousands of equations of state. Every one EOS follows causality, attaches at densities to the familiar EOS of neutron star crusts, is reserved by experimental and theoretical studies of the symmetry properties of matter near the nuclear saturation density, and satisfies the observational constraint for the maximum mass of a neutron star, $m \geq 2M_{\odot}$. Since the compactness parameter is defined by $C \equiv m/R$, the star's tidal deformability Λ is related to compactness parameter C by Eq. (14) and gives $\Lambda \propto C^{-5}$ .However, a better description $\Lambda \propto C^{-6}$ is provided by for moderate masses because the behavior $k_2 \propto C^{-1}$ is observed for a wide variety of EOS in the mass range $\left(1.1M_{\odot} \leq M \leq 1.6M_{\odot}\right)$.This mass range is the predictable range if perceived double neutron star binaries are typical merger candidates. But $m \to 0, k \to 0$ so that k_2 is related to C with positive power. So, in this relevant range the important result become

$$\Lambda = aC^{-6} \tag{24}$$

where $a = (2/3)k_2 = 0.0093 \pm 0.0007$ bounds the results for $1.1M_{\odot} \leq M \leq 1.6M_{\odot}$ .The C-dependence of Λ has interesting consequences for the binary deformability $\tilde{\Lambda}$, equation (13). An immediate result motivated by the observation with piecewise polytropes that $\Lambda \approx aC^{-6}$ and $R_1 \approx R_2$ is

$$\Lambda_1 \approx q^6 \Lambda_2 \tag{25}$$

The above correlation is used in the analysis of the gravitational wave signal from GW 170817. Then the binary tidal deformability $\tilde{\Lambda}$ with different q values is shown in Figure (2). The common equation of state (EOS) constraint allows to show that the binary tidal deformability $\tilde{\Lambda}$ is essentially a function of the chirp mass M, the common radius R , and the mass ratio q[4], but that its dependence on q is very weak. Substituting the expressions $\Lambda \approx aC^{-6}$ and $R_1 \approx R_2 \approx R$ into Equation (23)

$$\tilde{\Lambda} = \frac{16}{13}\frac{(12q+1)\Lambda_1 + (12+q)q^4\Lambda_2}{(1+q)^5}$$

$$\tilde{\Lambda} = \frac{16}{13}\frac{(12q+1)\Lambda_1 + (12+q)q^{-2}\Lambda_1}{(1+q)^5}$$

$$\tilde{\Lambda} = \frac{16}{13}\frac{\left((12q+1) + (12+q)q^{-2}\right)\Lambda_1}{(1+q)^5}$$

$$\tilde{\Lambda} = \frac{16}{13}\frac{\left(q^2(12q+1) + 12 + q\right)\Lambda_1}{q^2(1+q)^5}$$

$$\tilde{\Lambda} = \frac{16}{13}\frac{(1+q)\left(12q^3 + q^2 + 12 + q\right)\Lambda_1}{q^2(1+q)^5}$$

$$\tilde{\Lambda} = \frac{16}{13}\frac{\left(12q^2 - 11q + 12\right)\Lambda_1}{q^2(1+q)^4}$$

$$\tilde{\Lambda} = \frac{16}{13}\frac{\left(12q^2 - 11q + 12\right)}{q^2(1+q)^4}\left(\frac{2}{3}k_2\right)\left(\frac{R}{M}\right)^6$$

$$\tilde{\Lambda} = \frac{16}{13} a' \left(\frac{R}{M} \right)^6 \frac{\left(12q^2 - 11q + 12\right)}{q^2 \left(1+q\right)^4}$$

$$\tilde{\Lambda} = \frac{16}{13} a' \left(\frac{R}{M} \right)^6 f\left(q\right) \tag{26}$$

where $f\left(q\right) = \dfrac{\left(12q^2 - 11q + 12\right)}{q^2 \left(1+q\right)^4}$ which is very weakly dependent on q .Then Equation (26) becomes

$$\tilde{\Lambda} = \frac{16}{13} a' \left(\frac{R}{M} \right)^6 \tag{27}$$

where $a' = 0.0042 \pm 0.0004$ bounds the results for $1.1M_{\odot} \le M \le 1.6M_{\odot}$. In this research the masses of binary neutron are taken as $m_1 \ge m_2$, $q \le 1$. Each equation of state in the piecewise polytrope scheme can compute $\tilde{\Lambda}$ for all stellar pairs along the corresponding to $M - R$. Then, Figure 4 show the results of $\tilde{\Lambda}$, where equations of state are identified by their corresponding value of $R_{1.4}$, the radius of a $1.4M_{\odot}$ star[5].

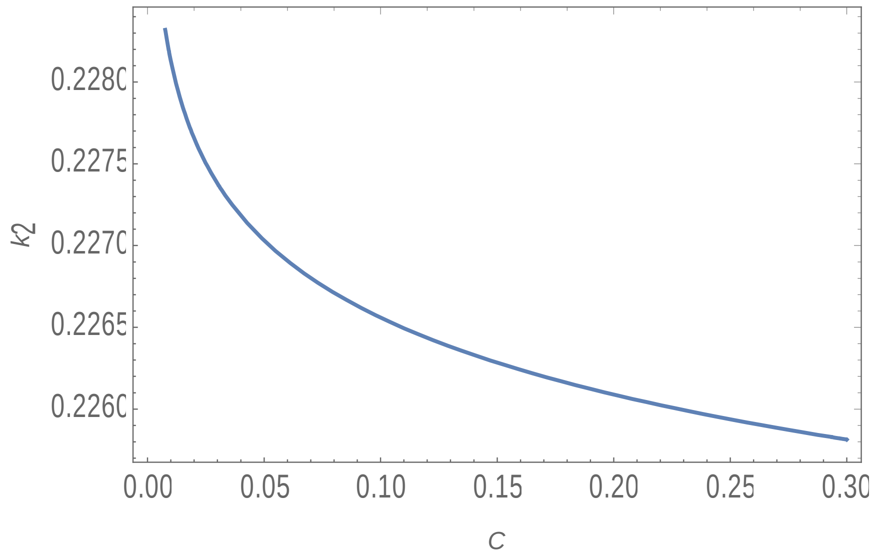

Figure 1 Dimensionless Love number (k_2) as a function of Compactness (C)

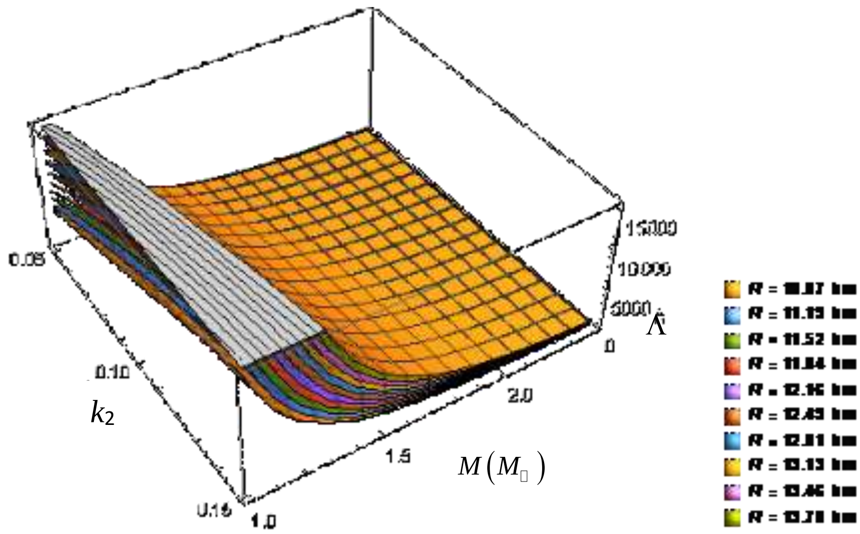

Figure 2 3D visualization of tidal deformity Λ

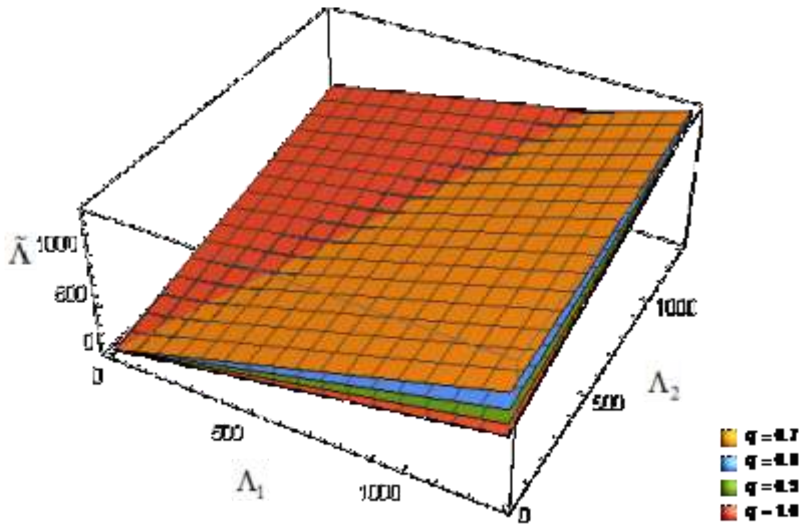

Figure 3 3D visualization of tidal deformity $\tilde{\Lambda}$ with Λ_1 and Λ_2

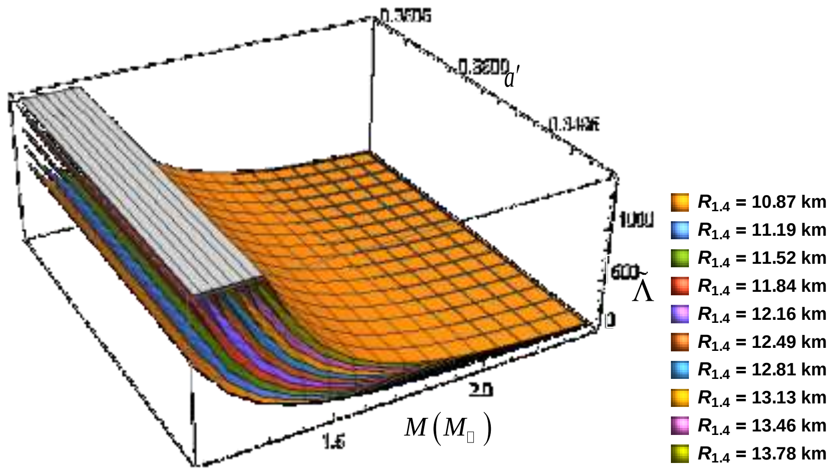

Figure 4 3D visualization of tidal deformity $\tilde{\Lambda}$

Concluding Remarks

In the present research, the tidal deformability of neutron star Λ and the dimensionless binary tidal deformability $\tilde{\Lambda}$ have been studied with the Love number k_2 and the radius of neutron stars. So, the corresponding tidal Love number is firstly calculated and the tidal deformability of static neutron star is calculated. So, tidal Love number (k_2) as a function of compactness (C) is shown in Figure 1. From this figure, the numerical range of Love number for polytrope equation of state are obtained. By using this range, tidal deformability as a function of mass for physically realistic polytropes is shown by Figure (2). In this figure, TOV integration with each EOS parameter set results in a series of values that are shown as points colored by their radii R. There are well-defined upper and lower bounds for Λ (M), with the upper (lower) bound defined by the stars with the largest (smallest) radii. The lower bound for $\Lambda(M)$ is an important constraint that should be taken into account in gravitational waveform modeling of BNS mergers. The dimensionless binary tidal deformability deformity $\tilde{\Lambda}$ with Λ_1 and Λ_2 is demonstrated in Figure (3) with $q \leq 1$. Then, for each equation of state in the piecewise polytrope scheme, one can compute $\tilde{\Lambda}$ for all stellar pairs along the corresponding to mass and radii. The results are displayed in Figure (4) which is similar to Figure (2), except that the dimensionless binary tidal deformability as a function of chirp mass M. Finally, in the case of binary neutron star system, many situations are possible, depending on the mass of components and EOS of matter. Besides, system with larger masses and less deformable matter result to prompt collapse to black hole after the merger. Binaries with smaller masses and more deformable matter lead to the formation of an unstable, possibly long-lived remnant.

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